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Nonlinear Enhancement of Weak Signals Using Optimization Theory

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Nonlinear Enhancement of Weak Signals Using Optimization Theory *

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Abstract—Stochastic resonance (SR) is a phenomenon that performance of the nonlinear system can be improved with the addition of optimal amount of noise. Stochastic resonance has been increasingly used for signal processing. The output of the nonlinear bistable dynamic system can be used to restore the weak input signal corrupted by white Gaussian noise, if the similarity between the input signal and the output can be maximized. This paper will first use the optimization theory to show that the normalized power norm (C_1) describing the similarity will reach a larger maximum when tuning both the system parameters and noise intensity, compared with that of only adjusting noise intensity (classical stochastic resonance) or only adjusting system parameters (parameter-tuning stochastic resonance). Then, a practical fast-converging optimization algorithm is mentioned to search the optimal system parameters and noise intensity. Finally, computer simulations are performed to verify this proposal and demonstrate its application in signal processing.

Index Terms—Optimization, Stochastic Resonance, Signal Processing

I. INTRODUCTION

Noise is usually thought to be annoying and should be removed from the system. In some nonlinear systems, however, the addition of some extra amount of noise has been shown to be helpful. This phenomenon is called Stochastic resonance (SR)[1][2], and only exists in certain nonlinear systems. For these systems, the synchronization between the input signal and the noise will happen when the noise intensity is adjusted properly. In these cases, the system performance, such as the output signal-to-noise ratio and mutual information, will benefit from the noise. The improvement of the system performance can be maximized if the noise intensity is adjusted to an optimal level. This phenomenon was first revealed by Benzi in 1981 to explain the periodically recurrent ice ages [3]. Since then, stochastic resonance has been continuously attracting considerable attention of researchers. Basically, the stochastic resonance involves four elements: nonlinear system, information-carrying

input signal, noise, and performance measure[8]. Many kinds of nonlinear systems have been shown to yield stochastic resonance phenomenon, such as the static systems[4] and dynamic systems[1]. The input signal can be periodic[1] or aperiodic[5]. The classical stochastic resonance requires the input signal to be subthreshold signal[1]. Recently, it was found that the input signals can be arbitrary and are not limited to weak signals. This is characterized by the suprathreshold stochastic resonance[6]. Not only the white Gaussian noise[1], but also the colored[9] and non-Gaussian noise[10], can generate a stochastic resonance effect. In order to describe the stochastic resonance more exactly, many quantifiers have been proposed as the performance measures, such as signal-to-noise ratio[1], power norm[5], and mutual information[7]. Stochastic resonance has found applications in many different areas, such as noise enhanced tactile sensation[11], the application of suprathreshold stochastic resonance to cochlear implant coding[12]. Another important application is in signal processing. It has been used for signal detection[13], signal transmission[15], signal estimation[17], and image processing[8]. The detector based on stochastic resonance can improve the robustness of the detector and its performance can compare with the locally optimum detectors(LOD)[14]. When the information is transmitted through a large parallel summing array, the noise can enhance performance up to approximately half the theoretical noiseless channel capacity[16]. The Bayesian estimator using stochastic resonance technique will achieve the minimum of the mean square estimation error when estimating the frequency of a periodic signal corrupted by a phase noise[17]. In order to make the noise useful, the stochastic resonance effect should be realized. For the traditional stochastic resonance, the stochastic resonance is realized by adjusting noise intensity[1]. Recently, the parameter-tuning stochastic resonance shows that tuning system parameters is a better method to realize stochastic resonance in some situations, especially when the initial input noise intensity is already beyond the resonance region[18][19][20]. The chosen performance measure will reach a higher/lower maximum/minimum, compared with that by adjusting noise

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intensity. This paper will apply optimization theory to show that the maximal normalized power norm of the bistable double-well system can be further increased by tuning system parameters and noise intensity at the same time, compared with that by parameter-tuning stochastic resonance and that by classical stochastic resonance. The normalized power norm is the performance measure describing the similarity between the input signal corrupted by white Gaussian noise and the output of this nonlinear system. The increase of the similarity between the input and the output by this scheme will benefit the restoration of the noisy weak input signal and has potential applications in signal processing.

The rest of the paper is organized as follows. Section II will describe the nonlinear bistable system and the related performance measure. In Section III, we will show it is possible to further enhance the stochastic resonance effect with tuning system parameters and noise intensity by working with a more general nonlinear bistable system. For the purpose of practical implementation, Section IV will introduce the optimization algorithm to search for the optimal system parameters and noise intensity. Section V will focus on the application of this scheme in signal processing. Finally, Section VI concludes the paper.

II. NONLINEAR BISTABLE DOUBLE-WELL STOCHASTIC RESONANCE SYSTEM

The nonlinear bistable double-well system can be expressed in the following equation[5]

$$\frac{dx}{dt} = -\frac{\partial U}{\partial x} + \xi(t), \quad (1)$$

where $U(x)$ is the potential function.

The symmetric potential function with a fluctuating barrier is given by[5]

$$U(x) = -[A - S(t)]\frac{x^2}{2} + \frac{x^4}{4}, \quad (2)$$

where A is positive and is taken as a tuning parameter in this paper. $S(t)$ is the aperiodic input signal with zero-mean average. $\xi(t)$ is white Gaussian noise with zero mean and autocorrelation of $\langle \xi(t)\xi(s) \rangle = 2D\delta(t-s)$. The angular brackets denote the ensemble average.

In order to demonstrate the stochastic resonance effect, the cross-correlation measures (power norm C_0 , and normalized power norm C_1) are taken as the performance measures

$$C_0 = \max\{\overline{S(t)R(t+\tau)}\}, \quad (3)$$

$$C_1 = \frac{C_0}{[\overline{S^2(t)}]^{1/2} \{[\overline{R(t)} - \overline{R(t)}]^2\}^{1/2}}, \quad (4)$$

where $R(t)$ is used as the system response characterized by mean transition rate of the system. The overbar denotes an average over time. τ is a time lag.

The cross-correlation describes the similarity between the input signal corrupted by the white Gaussian noise and the system output. Usually, it is hard to find an explicit expression for the above cross-correlation. If the input signal is both

weak, i.e., $\overline{S(t)^2} \ll A^2$, and also of Gaussian-distribution, the ensemble averaged power norm $\langle C_0 \rangle$ and the ensemble averaged normalized power norm $\langle C_1 \rangle$ can be approximated by[5]

$$\langle C_0 \rangle \simeq Q_0 \Delta_0 \exp[-\Theta_0 + \Delta_0^2 \overline{S^2(t)}/2] \overline{S^2(t)}, \quad (5)$$

$$\langle C_1 \rangle \simeq \frac{\Delta_0 [\overline{S^2(t)}]^{1/2}}{(e^{\Delta_0^2 \overline{S^2(t)}} - 1 + \sigma(D) Q_0^{-2} e^{(2\Theta_0 - \Delta_0^2 \overline{S^2(t)})})^{1/2}}, \quad (6)$$

where:

$$\sigma(D) = K_1 \langle \overline{R(t)} \rangle, \quad \langle \overline{R(t)} \rangle \simeq Q_0 \exp[-\Theta_0 + \Delta_0^2 \overline{S^2(t)}/2],$$

$$Q_0 = K_0 A / \sqrt{2\pi}, \quad \Theta_0 = A^2 / 4D, \quad \Delta_0 = A / 2D.$$

$\langle \overline{R(t)} \rangle$ is the ensemble-averaged escape rate. Also, the angular brackets denote an ensemble average.

It was revealed in [5] that this performance measure or the similarity between input and output will be maximized when an optimal amount of additional noise is added into the system. This is termed aperiodic stochastic resonance (ASR). Now, we will discuss the method in which the similarity can be further enhanced so that the corrupted input signal can be better restored.

III. ENHANCEMENT OF STOCHASTIC RESONANCE EFFECT

Usually, the stochastic resonance effect is realized either by adjusting noise intensity (classical stochastic resonance) or by tuning system parameters (parameter-tuning stochastic resonance), but not both. In some cases, the parameter-tuning method is better than the classic method. Intuitively, the stochastic resonance effect can be further enhanced if both the system parameters and the noise intensity are adjusted at the same time. This, however, is not always true, even if the performance measure is affected by both the system parameters and the noise intensity. In [21], the bit error rate (BER) will be minimized if the noise intensity is not adjusted and fixed at the initial level, while the system parameter is tuned to the optimal value. In our recent paper[22], we demonstrate that, for the bistable double-well dynamic system with Gaussian-distributed input signal and fluctuating barrier, it is possible to further increase the normalized power norm $\langle C_1 \rangle$ by tuning system parameter and noise intensity at the same time.

In [22], we require that two additional parameters should be introduced into the system, in order to make this mechanism possible for different weak input signals. These two new system parameters in [22], however, do not have direct physical meaning. This makes the mechanism of tuning system parameters and noise intensity a little hard to understand. Now, we will change to a new system model in which these system parameters will have direct physical meaning. Based on this model, we will also prove that this mechanism is true not only for the Gaussian-distributed input signals, but also for the general weak input signals.

The new potential function of this nonlinear system is modified as

$$U(x) = -[A - S(t)]\frac{x^2}{2} + \frac{x^4}{4X_b^2}, \quad (7)$$

where X_b is one of the two new system parameters.

By introducing another new system parameter τ_a , the nonlinear dynamic system is now described by the following equation

$$\tau_a \dot{x}(t) = [A - S(t)]x(t) - \frac{x^3(t)}{X_b^2} + \xi(t), \quad (8)$$

where the definitions of $S(t)$ and $\xi(t)$ are same as (1).

In this nonlinear system, the system parameters are τ_a , X_b and A . Parameter τ_a will affect the system response time and parameter X_b will affect the barrier height of the potential function of this system. Parameter A is used to shift the input signal. All these three system parameters have direct physical meaning and their influences on the the system performance measure $\langle C_1 \rangle$ are easier to understand, compared with the one used in [22].

From [5], we know the ensemble-averaged escape rate can be expressed as follow, if the parameter $\tau_a = 1$

$$\langle R(t) \rangle \simeq \frac{1}{2\pi} \sqrt{U''(x_{min})|U''(x_{max})|} e^{\frac{U(x_{min}) - U(x_{max})}{D}}, \quad (9)$$

where U is the potential function, x_{min} is one of the local minimizers and x_{max} is the local maximizer.

The new nonlinear system equation and the new potential function will affect the $\langle R(t) \rangle$. The method to derive the approximation of $\langle C_1 \rangle$ is similar to the one to derive (6). Also, for the general weak input signal case, we can derive the final approximation of normalized power norm $\langle C_1 \rangle$, if the condition $\Delta^2 \bar{S}(t)^2 \ll 1$ is met

$$\langle C_1 \rangle \simeq \frac{\Delta s}{(\Delta^2 s^2 + \frac{\Delta^4 s^4}{2} + \frac{K_1}{Q}(1 - \frac{\Delta^2 s^2}{2} + \frac{\Delta^4 s^4}{8})e^{c\tau_a Q \Delta})^{1/2}}, \quad (10)$$

where:

$$Q = \frac{K_0 A}{\sqrt{2} \tau_a \pi}, \quad \Theta = \frac{\tau_a X_b^2 A^2}{4D} = c\tau_a \Delta Q, \quad \Delta = \frac{\tau_a X_b^2 A}{2D}, \quad s = \sqrt{\bar{S}(t)^2}.$$

In order to investigate whether the stochastic resonance effect of the above nonlinear system can be further enhanced by tuning system parameters and noise intensity at the same time, we need to check whether the following constrained optimization problem has global maximizer

$$\max \langle C_1 \rangle, \quad (11)$$

subject to: $A > 0, s^2 \ll A^2, \Delta^2 s^2 \ll 1, D_0 \leq D \leq D_1$

The constrain of $\bar{S}(t)^2 \ll A^2$ comes from the requirement on weak input signal. Parameter A is positive and $[A - \bar{S}(t)]$ is also positive. In addition, $\Delta^2 s^2 \ll 1$ should be satisfied in order to make (10) valid. According to [5], the theoretic expression $\langle C_1 \rangle$ can still predict its real shape, even if the noise intensity is beyond the range of its validity. Also, we assume here that the noise cannot be removed. So, the only requirements on the noise intensity are that it cannot be less than its initial value D_0 , and it can not be arbitrarily large.

For this optimization problem, we will take parameters A and D as the optimal parameters, while parameter τ_a and X_b will be taken as the supporting parameters which are used to ensure the optimization problem (11) has solution, as shown later. In order to simplify the calculation, the direct optimal parameters of (11) are Δ and Q , which are in turn the functions of A and D .

We now prove that (11) has one and only one global maximizer. Here, we will first define the corresponding unconstrained optimization problem as

$$\max \langle C_1 \rangle. \quad (12)$$

Proposition 1: The unconstrained optimization problem (12) has one and only one pair of parameters (Q^*, Δ^*) satisfying the first-order necessary condition for a local maximizer.

Proof: According to the first-order necessary condition of this optimization problem (12), we have

$$\frac{\partial \langle C_1 \rangle}{\partial \Delta} = 0 \quad \text{and} \quad \frac{\partial \langle C_1 \rangle}{\partial Q} = 0. \quad (13)$$

Then, we can derive $c\tau_a \Delta Q = 1$, and

$$-s^4 \Delta^3 + c\tau_a e K_1 (1 + s^2 \Delta^2 / 2 - 3s^4 \Delta^4 / 8) = 0. \quad (14)$$

Let $f(\Delta) = -s^4 \Delta^3 + c\tau_a e K_1 (1 + s^2 \Delta^2 / 2 - 3s^4 \Delta^4 / 8). \quad (15)$

Obviously, there is at least one solution (Q^*, Δ^*) satisfying this first-order necessary condition, because

$$f(0) = c\tau_a e K_1 > 0 \quad \text{and} \quad f(+\infty) = -\infty. \quad (16)$$

Let $f_1(\Delta) = -s^4 \Delta^3, \quad (17)$
 $f_2(\Delta) = c\tau_a e K_1 (1 + s^2 \Delta^2 / 2 - 3s^4 \Delta^4 / 8). \quad (18)$

Function $f_1(\Delta)$ is a monotonically decreasing function. Function $f_2(\Delta)$ will first increase with Δ and then decrease to $-\infty$. From these facts, we can prove that the first-order necessary condition can only have one positive solution. ■

Proposition 2: The system parameter τ_a can continuously adjust (Q^*, Δ^*) satisfying the first-order necessary condition. ,

Proof: The system parameter τ_a affects $f_2(\Delta)$, but not $f_1(\Delta)$. From the special characteristics of these two functions, we can find out that the increase of τ_a will also increase the value of Δ^* satisfying (14). If τ_a is getting close to zero, Δ^* will also approach zero. From this, we complete the proof of this proposition. ■

Proposition 3: The unconstrained optimization problem (12) has one and only one local maximizer when the input is small and the system parameters τ_a and X_b are chosen properly.

Proof: From Proposition 1, we know the first-order necessary condition only has one solution (Q^*, Δ^*) . We now prove this solution will also satisfy the second-order sufficient condition for a local maximizer, that is the Hessian matrix is negative definite at the point (Q^*, Δ^*) .

According to Proposition 2, the system parameter τ_a can be adjusted properly so that the requirement $s^2\Delta^{*2} \ll 1$ can be satisfied. From this, we can get

$$\begin{aligned} & -c\tau_a e K_1 s^2 \Delta^{*2} + s^2 \Delta^* (-4 + 2s^2 \Delta^{*2}) \\ & -c\tau_a e K_1 s^2 \Delta^{*2} (2 - 3s^2 \Delta^{*2}/4) < 0, \end{aligned} \quad (19)$$

and

$$-s^4 \Delta^4 / 8 + (-1 + s^2 \Delta^{*2} / 2) < 0. \quad (20)$$

From (14), (19), and (20), it follows that $\frac{\partial^2 \langle C_1 \rangle}{\partial \Delta^2}$, $\frac{\partial^2 \langle C_1 \rangle}{\partial Q^2}$, and $\frac{\partial^2 \langle C_1 \rangle}{\partial \Delta \partial Q}$ are all negative at $\Delta = \Delta^*$, and $Q = Q^*$.

The Hessian matrix is defined as

$$\begin{pmatrix} \frac{\partial^2 \langle C_1 \rangle}{\partial \Delta^2} & \frac{\partial^2 \langle C_1 \rangle}{\partial \Delta \partial Q} \\ \frac{\partial^2 \langle C_1 \rangle}{\partial Q \partial \Delta} & \frac{\partial^2 \langle C_1 \rangle}{\partial Q^2} \end{pmatrix}$$

To prove the Hessian matrix is negative definite, we need to verify its determinant value is positive.

At $\Delta = \Delta^*$ and $Q = Q^*$, the numerator of this Hessian matrix determinant value can be simplified as:

$$\begin{aligned} & s^4 \Delta^{*3} (2 - 2s^2 \Delta^{*2}) + c\tau_a e K_1 s^4 \Delta^{*4} \left(\frac{14}{8} - \frac{9s^2 \Delta^{*2}}{8} \right) \\ & + (s^4 \Delta^{*3} + \frac{3c\tau_a e K_1 s^4 \Delta^{*4}}{8} - \frac{c\tau_a e K_1 s^2 \Delta^{*2}}{2}) \\ & + \frac{s^8 \Delta^{*7}}{2} + \frac{15c\tau_a e K_1 s^8 \Delta^{*8}}{64}, \end{aligned} \quad (21)$$

Its numerator will be positive, if $s^2 \Delta^{*2} \ll 1$. Also, its denominator is positive. From the standard test on negative-definiteness of a symmetric matrix, it follows the Hessian matrix is negative definite. This completes the proof of Proposition 3. ■

Proposition 4: The constrained optimization problem (11) with small input has one and only one global maximizer, if the system parameters τ_a and X_b are chosen properly.

Proof: According to Proposition 3, (12) has one and only one local maximizer (Q^*, Δ^*) . Also, from Proposition 2, Δ^* can be continuously adjusted by the system parameter τ_a such that $s^2 \Delta^{*2} \ll 1$. In this case, the requirement for $s^2 \ll A^{*2}$ will also be satisfied and system parameter A^* will be positive, because of $A^* = 2/\Delta^*$.

The constraints on the noise intensity can also be satisfied by tuning system parameters τ_a and X_b , because of $D^* = \tau_a X_b^2 / \Delta^{*2}$. So, the constrained optimization problem (11) has one and only one local maximizer.

It is obvious that the only local maximizer is also the global maximizer of (11). This completes the proof of this proposition. ■

Proposition 4 reveals that the results of [22] can be extended to the more general weak input signal case, when the nonlinear system model with more physical meaning is adopted. This system's maximal normalized power norm $\langle C_1 \rangle$ can be further increased with this scheme, compared with that of either tuning system parameters or adjusting noise intensity.

IV. OPTIMIZATION ALGORITHM

In order to meet performance requirements for some tasks, such as high-speed target detection, it is important to develop a fast-converging optimization algorithm to search the optimal system parameters and noise intensity.

The optimal parameters (A^*, D^*) of the constrained optimization problem (11) can be derived from the values (Q^*, Δ^*) satisfying (14). We will use Newton's method for nonlinear equations to solve (14), because $\nabla f(\Delta^*)$ is non-singular for the weak input signals. According to [23], this method will reach a local Q -quadratic convergence, if the initial value is selected properly.

Our algorithm provides a method to estimate the initial value for the Newton algorithm to ensure the fast convergence.

In this algorithm, the optimal parameters are Q , Δ , τ_a and X_b . The optimal value (A^*, D^*) can be derived from these values. X_b will only affect the optimal noise intensity and should be chosen to ensure (11) has solution. Here, we assume τ_a is also adjustable. From (10), we notice that the increase of τ_a will decrease the value of $\langle C_1 \rangle$, if other parameters are fixed. This means that there will be no local maximizer for τ_a , it can only take the extremum.

The main idea of this algorithm is that the tables describing the relationship between Δ^* , input signal, and τ_a will be used to estimate the initial values for the given input signal. These tables are constructed off-line. The estimated initial values are close enough to the optimal values to ensure the fast convergence of this algorithm. The details of the algorithm can refer to [22]. The only changes are that the "too large" condition means $D < D_1$ for the first case, and it means $D < D_0$ or the requirement $s^2 \Delta^{*2} \ll 1$ is not met for the second case.

V. ENHANCEMENT OF WEAK SIGNALS

The traditional method to restore the weak signal corrupted by noise will try to remove noise from the signal. The method based on stochastic resonance (SR), on the contrary, can improve the performance measure, such as signal-to-noise ratio, with the addition of an extra amount of noise. A critical task in developing the SR-based signal processor is how to realize stochastic resonance and how to enhance stochastic resonance effect. In this paper, the bistable double-well dynamic system acts as a nonlinear filter to restore the

input signal corrupted by noise. The normalized power norm $\langle C_1 \rangle$ is adopted as the performance measure. Obviously, a larger $\langle C_1 \rangle$ value means that the input signal and the output signal are more similar. This, in turn, means that the corrupted input signal is better restored by this nonlinear system. In this case, the mechanism to further improve the maximal $\langle C_1 \rangle$ with tuning system parameters and noise intensity will have practical usage. It can better restore the signal from noise than the traditional SR method, which only adjusts the noise intensity.

This SR-based nonlinear filters with tuning system parameters and noise intensity can find applications at least in two situations. The first case is when the traditional denoising filter cannot completely remove the noise from the signal. In this case, the SR-based nonlinear filter is a good candidate to be used as a post-processor to further improve performance. The second case is when this nonlinear SR dynamic system is a part of the whole system under investigation. In this case, tuning the system parameters and noise intensity to maximize the enhancement of the weak input signal corrupted by noise will benefit the rest of the system. For example, it will be easier to process an input with higher signal-to-noise ratio.

In order to demonstrate this mechanism's better enhancement of weak signal compared with the traditional stochastic resonance method, simulations are performed. The first simulation is to directly compare the maximal $\langle C_1 \rangle$ reached by three different methods: (1) adjusting system parameters and noise at the same time; (2) only adjusting system parameters; (3) only adjusting noise intensity. The simulation result is shown in Fig. 1. From this figure, it is obvious that the mechanism proposed in this papers gives the best performance, especially for the weak input signal case.

Now, we will directly deal with the system output $x(t)$ and compare their waveforms, in order to better demonstrate the nonlinear enhancement of the weak input signal by this method. Fig. 2 is the simulation model. The original input signal and its corrupted noisy signal are displayed in Fig. 3, while Fig. 4 shows some of the system output $x(t)$ simulation results for the same input signal with different values of system parameter A and noise intensity D .

From this simulation, it is easy to notice that the similarity between input and output, or the denoising effect, is greatly affected by the choices of the system parameter A and noise intensity D . It will be maximized for the properly chosen A and D . This can be even more obvious, if a similar signal recovery method as in [20] is adopted to further recover input $S(t)$ from $x(t)$.

VI. CONCLUSION

This paper first reveals that the results of [22] can be extended to a modified bistable double-well nonlinear dynamic system with more general weak input signals. In this

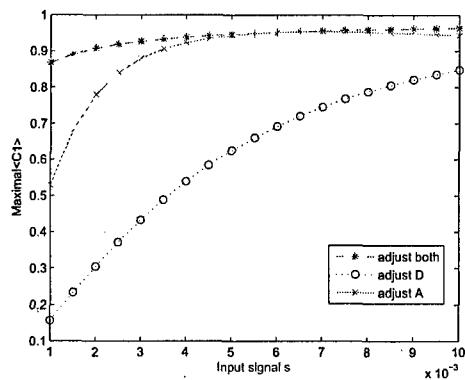


Fig. 1. Comparison of Maximal $\langle C_1 \rangle$

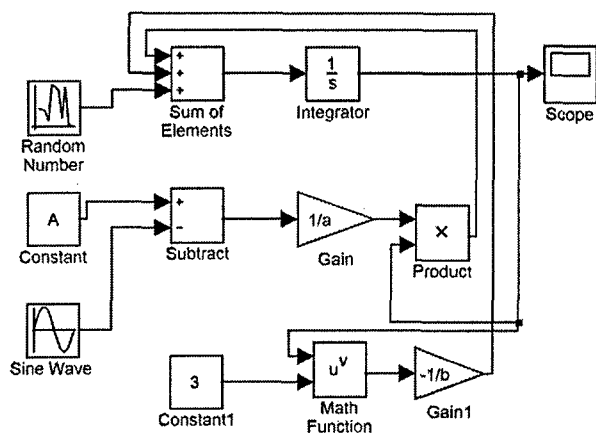


Fig. 2. Simulation Model

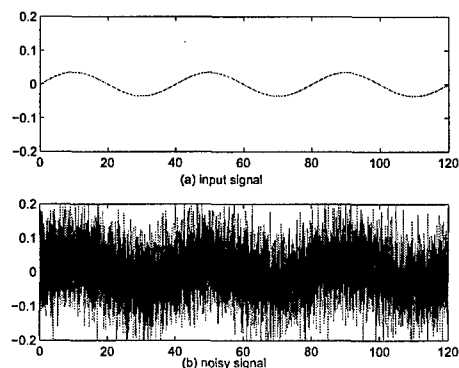


Fig. 3. Original & Noisy Signals

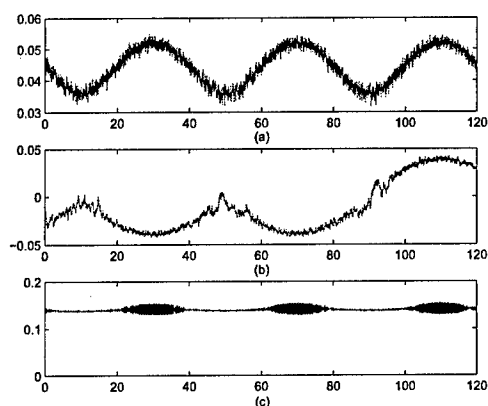


Fig. 4. System Output for Different A and D (a) $A=0.1$, $\sigma=0.065$ (b) $A=0.04$, and $\sigma=0.11$ (c) $A=0.99$, and $\sigma=0.0325$

nonlinear system, the system parameters have more physical meaning and the tuning of these system parameters is easier to understand. Then, the mechanism of tuning system parameters and noise intensity at the same time is applied to the recovery of weak signal from noise. This nonlinear denoising filter based on stochastic resonance plays a unique role in the nonlinear enhancement of weak signals corrupted by noise, compared with traditional denoising filters. Our future work will be directed at extending the application areas of this SR-based filter and comparing its performance with other filters in these applications.

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14. ABSTRACT Stochastic Resonance (SR) is a phenomenon that performance of the nonlinear system can be improved with the addition of optimal amount of noise. Stochastic resonance has been increasingly used for signal processing. The output of the nonlinear bistable dynamic system can be used to restore the weak input signal corrupted by white Gaussian noise, if the similarity between the input signal and the output can be maximized. This paper will first use the optimization theory to show that the normalized power norm (C1) describing the similarity will reach a larger maximum when tuning both the system parameters and noise intensity, compared with that of only adjusting noise intensity (classical stochastic resonance) of only adjusting system parameters (parameter-tuning stochastic resonance). Then, a practical fast-converging optimization algorithm is mentioned to search the optimal system parameters and noise intensity. Finally, computer simulations are performed to verify this proposal and demonstrate its application in signal processing.					
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Nonlinear Enhancement of Weak Signals Using Optimization Theory *

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Abstract—Stochastic resonance (SR) is a phenomenon that performance of the nonlinear system can be improved with the addition of optimal amount of noise. Stochastic resonance has been increasingly used for signal processing. The output of the nonlinear bistable dynamic system can be used to restore the weak input signal corrupted by white Gaussian noise, if the similarity between the input signal and the output can be maximized. This paper will first use the optimization theory to show that the normalized power norm ($C1$) describing the similarity will reach a larger maximum when tuning both the system parameters and noise intensity, compared with that of only adjusting noise intensity (classical stochastic resonance) or only adjusting system parameters (parameter-tuning stochastic resonance). Then, a practical fast-converging optimization algorithm is mentioned to search the optimal system parameters and noise intensity. Finally, computer simulations are performed to verify this proposal and demonstrate its application in signal processing.

Index Terms—Optimization, Stochastic Resonance, Signal Processing

I. INTRODUCTION

Noise is usually thought to be annoying and should be removed from the system. In some nonlinear systems, however, the addition of some extra amount of noise has been shown to be helpful. This phenomenon is called Stochastic resonance (SR)[1][2], and only exists in certain nonlinear systems. For these systems, the synchronization between the input signal and the noise will happen when the noise intensity is adjusted properly. In these cases, the system performance, such as the output signal-to-noise ratio and mutual information, will benefit from the noise. The improvement of the system performance can be maximized if the noise intensity is adjusted to an optimal level. This phenomenon was first revealed by Benzi in 1981 to explain the periodically recurrent ice ages [3]. Since then, stochastic resonance has been continuously attracting considerable attention of researchers. Basically, the stochastic resonance involves four elements: nonlinear system, information-carrying

input signal, noise, and performance measure[8]. Many kinds of nonlinear systems have been shown to yield stochastic resonance phenomenon, such as the static systems[4] and dynamic systems[1]. The input signal can be periodic[1] or aperiodic[5]. The classical stochastic resonance requires the input signal to be subthreshold signal[1]. Recently, it was found that the input signals can be arbitrary and are not limited to weak signals. This is characterized by the suprathreshold stochastic resonance[6]. Not only the white Gaussian noise[1], but also the colored[9] and non-Gaussian noise[10], can generate a stochastic resonance effect. In order to describe the stochastic resonance more exactly, many quantifiers have been proposed as the performance measures, such as signal-to-noise ratio[1], power norm[5], and mutual information[7]. Stochastic resonance has found applications in many different areas, such as noise enhanced tactile sensation[11], the application of suprathreshold stochastic resonance to cochlear implant coding[12]. Another important application is in signal processing. It has been used for signal detection[13], signal transmission[15], signal estimation[17], and image processing[8]. The detector based on stochastic resonance can improve the robustness of the detector and its performance can compare with the locally optimum detectors(LOD)[14]. When the information is transmitted through a large parallel summing array, the noise can enhance performance up to approximately half the theoretical noiseless channel capacity[16]. The Bayesian estimator using stochastic resonance technique will achieve the minimum of the mean square estimation error when estimating the frequency of a periodic signal corrupted by a phase noise[17]. In order to make the noise useful, the stochastic resonance effect should be realized. For the traditional stochastic resonance, the stochastic resonance is realized by adjusting noise intensity[1]. Recently, the parameter-tuning stochastic resonance shows that tuning system parameters is a better method to realize stochastic resonance in some situations, especially when the initial input noise intensity is already beyond the resonance region[18][19][20]. The chosen performance measure will reach a higher/lower maximum/minimum, compared with that by adjusting noise

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